Supplement

On the structure of stationary solutions of the Navier-Stokes-equations

Peter Wittwer*

Département de Physique Théorique

Université de Genève, Switzerland

peter.wittwer@physics.unige.ch

April 12, 2002

Abstract

This paper is a supplementary section to [1]. We show that without any additional hypothesis the main result in [1] (Theorem 1) can be considerably strengthened.

Note: *This paper can not be read independently of [1]. The numbering of equations, theorems and propositions as well as cross-references used here have to be understood as if this paper were an additional section to [1].*

6 Improved version of Theorem 1

In this supplementary section we show that, without changing the hypotheses, Theorem 1 can be replaced by the following theorem.

Theorem 18 (Improved version of Theorem 1) *Let Σ and Ω as defined above. Then, for each $\mathbf{u}_* = (u_*, v_*)$ in a certain set of vector fields $\mathcal{S}$ to be defined later on, there exist a (locally unique) vector field $\mathbf{u} = \mathbf{u}_\infty + (u, v)$ and a function p satisfying the Navier-Stokes equations (1) and (2) in Ω and the boundary conditions (3) and (4). Furthermore,*

$$\lim_{x\to\infty} x^{1/2} \left(\sup_{y\in\mathbf{R}} |(u-u_0)(x,y)| \right) = 0 \ , \tag{69}$$

$$\lim_{x\to\infty} x \left(\sup_{y\in\mathbf{R}} |(v-v_0)(x,y)| \right) = 0 \ , \tag{70}$$

where

$$u_0(x,y) = \frac{c}{2\sqrt{\pi}} \frac{1}{\sqrt{x}} e^{-\frac{y^2}{4x}} + \frac{d}{\pi} \frac{x}{x^2+y^2} \ , \tag{71}$$

$$v_0(x,y) = \frac{c}{4\sqrt{\pi}} \frac{y}{x^{3/2}} e^{-\frac{y^2}{4x}} + \frac{d}{\pi} \frac{y}{x^2+y^2} \ , \tag{72}$$

*Supported in part by the Fonds National Suisse.

with

$$c = \lim_{k \to 0^+} \int_{\mathbf{R}} e^{iky} \left(u_*(y) + iv_*(y)\right) dy \ , \tag{73}$$

$$d = \lim_{k \to 0^+} \int_{\mathbf{R}} e^{iky} \left(-iv_*(y)\right) \ dy \ , \tag{74}$$

and

$$\int_{\mathbf{R}} (u - u_0)(x, y) \ dy = 0 \ , \tag{75}$$

for all $x \geq 1$.

A proof of this theorem is given below. It will follow rather easily from Proposition 23, which contains improved versions of the inequalities (28), (29), (31) and (32) of Proposition 6.

Remark 19 *The integrals in (73), (74) and (75) have to be understood in the* (C, δ)*-sense, with* $0 < \delta \leq 1$ *(see e.g. [3], Theorem 15). Namely, let* $C(\delta, R) = (1 - |y|/R)^{\delta}$. *Then the exact versions of (73), (74) and (75) are (using in addition that* u_* *is even and* v_* *is odd),*

$$c = \left(\lim_{R \to \infty} \int_{-R}^{R} C(\delta, R) \ u_*(y) \ dy \right) - d \ ,$$

$$d = \lim_{k \to 0_+} \left(\lim_{R \to \infty} \int_{-R}^{R} C(\delta, R) \sin(ky) v_*(y) \ dy \right) \ ,$$

and

$$\lim_{R \to \infty} \int_{-R}^{R} C(\delta, R) \ (u - u_0)(y) \ dy = 0 \ .$$

Remark 20 *The terms multiplied by* d *in* u_0 *and* v_0 *are the velocity field of a source located at* $x = y = 0$. *We could replace these terms without changing the theorem by those of a source with the same amplitude but located at* $x = x_0 < 1$ *and* $y = 0$. *The terms produced by the difference of two such sources are bounded for* $x \geq 1$ *and vanish as* x *goes to infinity sufficiently rapidly for (69) and (70) to remain valid.*

Remark 21 *The source-term in (71) (term multiplied by* d*) is irrelevant in the sense that (69) is satisfied for any value of* d. *The equation (75), however, is only satisfied for* d *as given in (74).*

Remark 22 *The constant* c *is the amount of fluid transported within the wake. Indeed, it follows from (71) and (75) that, for any* $0 < \varepsilon < 1/2$ *and* $0 < \delta \leq 1$,

$$c = \lim_{x \to \infty} \int_{-x^{1/2+\varepsilon}}^{x^{1/2+\varepsilon}} C(\delta, x^{1/2+\varepsilon}) \ u(x, y) \ dy \ ,$$

whereas the total fluid flow through a vertical line is

$$\lim_{R \to \infty} \int_{-R}^{R} C(\delta, R) \ u(x, y) \ dy = d + c \ ,$$

independently of $x \geq 1$. *On physical grounds (see for example [2]) we expect that* $c < 0$ *for the case of a wake created by a moving body, since the fluid supposedly flows less fast within the wake than at infinity. This means that, in a reference frame attached to the moving body, fluid is transported within the wake towards the body. This fluid is then "radiated" away from the body by the source-like contribution to the velocity field (the terms multiplied by* d*). For the case of boundary conditions on the body and at infinity as given after equation (8) we therefore expect that* $d = -c/2$.

6.1 Improved versions of inequalities (28), (29), (31) and (32).

Proposition 23 *Let u_-^*, w_-^* and q as defined above. Let*

$$\Omega_-(k,t) = \left(\omega_-^*(k) - \frac{1}{\Delta}\int_1^t q(k,s)\ ds\right) e^{\Lambda_-(t-1)} \ , \tag{76}$$

$$U_-(k,t) = \left(u_-^*(k) - \frac{1}{2}\frac{i}{k}\int_1^t q(k,s)\ ds\right) e^{-|k|(t-1)} \ , \tag{77}$$

and let for, $\alpha' \geq 0$,

$$\widetilde{\mu}_{\alpha'}(k,t) = \frac{1}{1+\left(|k|t^{3/4}\right)^{\alpha'}} \ . \tag{78}$$

Then, we have the following bounds,

$$|\omega_-(k,t) - \Omega_-(k,t)| \leq \frac{\varepsilon}{t}\mu_{\alpha+1}(k,t) \ , \tag{79}$$

$$\left|-\frac{i}{k}\Lambda_+\left(\omega_-(k,t) - \Omega_-(k,t)\right)\right| \leq \frac{\varepsilon}{t^{1/2}}\mu_{\alpha+1}(k,t) \ , \tag{80}$$

$$|u_+(k,t)| \leq \frac{\varepsilon}{t^{3/4}}\mu_{\alpha+1}(k,t) + \frac{\varepsilon}{t^{1/2}}\widetilde{\mu}_{\alpha+1}(k,t) \ , \tag{81}$$

$$|u_-(k,t) - U_-(k,t)| \leq \frac{\varepsilon}{t^{3/4}}\mu_{\alpha+1}(k,t) + \frac{\varepsilon}{t^{1/2}}\widetilde{\mu}_{\alpha+1}(k,t) \ . \tag{82}$$

Proof. We first prove (81). We have that

$$\begin{aligned}\int_t^\infty e^{|k|(t-s)}\frac{ds}{s^{3/2}} &\leq \int_t^{t+t^{3/4}}\frac{ds}{s^{3/2}} + \int_{t+t^{3/4}}^\infty e^{|k|(t-s)}\frac{ds}{s^{3/2}} \\ &\leq \frac{\text{const.}}{t^{3/4}} + \frac{\text{const.}}{t^{1/2}}e^{-t^{3/4}|k|} \ ,\end{aligned}$$

and therefore

$$\begin{aligned}|u_+(k,t)| &\leq \varepsilon\mu_{\alpha+1}(k,t)\int_t^\infty e^{|k|(t-s)}\frac{ds}{s^{3/2}} \\ &\leq \frac{\varepsilon}{t^{3/4}}\mu_{\alpha+1}(k,t) + \frac{\varepsilon}{t^{1/2}}e^{-t^{3/4}|k|} \\ &\leq \frac{\varepsilon}{t^{3/4}}\mu_{\alpha+1}(k,t) + \frac{\varepsilon}{t^{1/2}}\widetilde{\mu}_{\alpha+1}(k,t)\end{aligned}$$

as claimed. To prove the bounds on ω_- and u_-, we first show that

$$\omega_-(k,t) = \Omega_-(k,t) - \frac{\Lambda_-}{\Delta}\int_1^t e^{\Lambda_-(t-s)}q_1(k,s,t)\ ds \ , \tag{83}$$

$$u_-(k,t) = U_-(k,t) + \frac{i}{2}\frac{|k|}{k}\int_1^t e^{-|k|(t-s)}q_1(k,s,t)\ ds \ , \tag{84}$$

where

$$q_1(k,s,t) = -\int_s^t q(k,\sigma)\ d\sigma \ .$$

Namely, integration by parts in (16) leads to

$$\omega_-(k,t) = \omega_-^*(k)e^{\Lambda_-(t-1)} - \frac{1}{\Delta}\left[e^{\Lambda_-(t-s)}q_1(k,s,t)\right]_{s=1}^{s=t} - \frac{\Lambda_-}{\Delta}\int_1^t e^{\Lambda_-(t-s)}q_1(k,s,t)\ ds \ ,$$

$$u_-(k,t) = u_-^*(k)e^{-|k|(t-1)} - \frac{1}{2}\frac{i}{k}\left[e^{-|k|(t-s)}q_1(k,s,t)\right]_{s=1}^{s=t} + \frac{1}{2}\frac{i}{k}|k|\int_1^t e^{-|k|(t-s)}q_1(k,s,t)\ ds \ ,$$

and (83) and (84) follow, since $q_1(k,t,t)=0$. From the bound (20) on q it follows, that for $1 \leq s \leq t$,

$$|q_1(k,s,t)| \leq \varepsilon|k|\mu_{\alpha+1}(k,s)\frac{t-s}{\left(\sqrt{t}+\sqrt{s}\right)\sqrt{ts}} \ . \tag{85}$$

We now prove the bounds (79) and (80). Namely, from (85) and the inequality

$$\int_1^{(t+1)/2} \frac{t-s}{(\sqrt{t}+\sqrt{s})\,\sqrt{ts}} ds \leq \text{const.} \left(\frac{t-1}{t}\right)^2 \sqrt{t}\ ,$$

we find using Proposition 24 that

$$\begin{aligned}
\left|\frac{\Lambda_-}{\Delta}\int_1^{(t+1)/2} e^{\Lambda_-(t-s)} q_1(k,s,t)\ ds\right| &\leq \frac{|\Lambda_-|}{\Delta}\varepsilon\,|k|\,\mu_{\alpha+1}(k,1)e^{\Lambda_-(t-1)/2}\left(\frac{t-1}{t}\right)^2\sqrt{t} \\
&\leq \varepsilon\mu_{\alpha+3/2}(k,1)e^{\Lambda_-(t-1)/2}\,|\Lambda_-|^{3/2}\left(\frac{t-1}{t}\right)^2\sqrt{t} \\
&\leq \frac{\varepsilon}{t}\left(\frac{t-1}{t}\right)^{1/2}\mu_{\alpha+3/2}(k,t)\ ,
\end{aligned}$$

and that

$$\begin{aligned}
\left|\frac{\Lambda_-}{\Delta}\int_{(t+1)/2}^t e^{\Lambda_-(t-s)} q_1(k,s,t)\ ds\right| &\leq \frac{\varepsilon}{\Delta}\,|k|\,\mu_{\alpha+1}(k,t)\int_{(t+1)/2}^t \frac{t-s}{(\sqrt{s}+\sqrt{t})\sqrt{ts}}e^{\Lambda_-(t-s)}\,|\Lambda_-|\ ds \\
&\leq \frac{\varepsilon}{\sqrt{\Delta}}\mu_{\alpha+1}(k,t)\int_{(t+1)/2}^t \frac{t-s}{(\sqrt{s}+\sqrt{t})\sqrt{ts}}e^{\Lambda_-(t-s)}\,|\Lambda_-|^{3/2}\ ds \\
&\leq \frac{\varepsilon}{t^{3/2}}\frac{1}{\sqrt{\Delta}}\mu_{\alpha+1}(k,t)\int_{(t+1)/2}^t (t-s)e^{\Lambda_-(t-s)}\,|\Lambda_-|^{3/2}\ ds \\
&\leq \frac{\varepsilon}{t^{3/2}}\frac{1}{\sqrt{\Delta}}\mu_{\alpha+1}(k,t)\int_{(t+1)/2}^t \frac{1}{\sqrt{t-s}}\ ds \\
&\leq \frac{\varepsilon}{t}\frac{1}{\sqrt{\Delta}}\mu_{\alpha+1}(k,t)\ ,
\end{aligned}$$

which proves (79). Similarly,

$$\begin{aligned}
\left|\frac{\Lambda_+}{k}\right|\left|\frac{\Lambda_-}{\Delta}\int_1^{(t+1)/2} e^{\Lambda_-(t-s)} q_1(k,s,t)\ ds\right| &\leq \varepsilon\mu_{\alpha+1}(k,1)e^{\Lambda_-(t-1)/2}\,|\Lambda_-|\left(\frac{t-1}{t}\right)^2\sqrt{t} \\
&\leq \frac{\varepsilon}{t^{1/2}}\left(\frac{t-1}{t}\right)\mu_{\alpha+1}(k,t)\ ,
\end{aligned}$$

and

$$\begin{aligned}
\left|\frac{\Lambda_+}{k}\right|\left|\frac{\Lambda_-}{\Delta}\int_{(t+1)/2}^t e^{\Lambda_-(t-s)} q_1(k,s,t)\ ds\right| &\leq \varepsilon\mu_{\alpha+1}(k,t)\int_{(t+1)/2}^t \frac{t-s}{(\sqrt{s}+\sqrt{t})\sqrt{ts}}e^{\Lambda_-(t-s)}\,|\Lambda_-|\ ds \\
&\leq \frac{\varepsilon}{t^{3/2}}\mu_{\alpha+1}(k,t)\int_{(t+1)/2}^t (t-s)e^{\Lambda_-(t-s)}\,|\Lambda_-|\ ds \\
&\leq \frac{\varepsilon}{t^{1/2}}\mu_{\alpha+1}(k,t)\ ,
\end{aligned}$$

which proves (80). Finally, we prove (82). For $1 \leq t \leq 2$ we have that

$$\begin{aligned}
\left|\int_1^t e^{-|k|(t-s)} q_1(k,s,t)\ ds\right| &\leq \varepsilon\mu_{\alpha+1}(k,1)\int_1^t e^{-|k|(t-s)}\,|k|\,\frac{t-s}{(\sqrt{s}+\sqrt{t})\sqrt{ts}}ds \\
&\leq \frac{\varepsilon}{t}\mu_{\alpha+1}(k,1)\int_1^t e^{-|k|(t-s)}\,|k|\,(t-s)\,ds \\
&\leq \varepsilon\mu_{\alpha+1}(k,1) \leq \frac{\varepsilon}{t^{1/2}}\mu_{\alpha+1}(k,t)\ ,
\end{aligned}$$

and for $t > 2$ we have that

$$\begin{aligned}\left|\int_1^{t-(t-1)^{3/4}} e^{-|k|(t-s)} q_1(k,s,t)\ ds\right| &\leq \varepsilon\mu_{\alpha+1}(k,1)\int_1^{t-(t-1)^{3/4}} e^{-|k|(t-s)}\,|k|\,\frac{t-s}{(\sqrt{s}+\sqrt{t})\sqrt{ts}}ds\\
&\leq \frac{\varepsilon}{t}\mu_{\alpha+1}(k,1)e^{-|k|\frac{(t-1)^{3/4}}{2}}\int_1^t e^{-|k|\frac{(t-s)}{2}}\frac{|k|\,(t-s)}{s^{1/2}}ds\\
&\leq \frac{\varepsilon}{t}\mu_{\alpha+1}(k,1)e^{-|k|\frac{(t-1)^{3/4}}{2}}\int_1^t \frac{ds}{s^{1/2}}\\
&\leq \frac{\varepsilon}{t^{1/2}}\mu_{\alpha+1}(k,1)e^{-|k|\frac{(t-1)^{3/4}}{2}} \leq \frac{\varepsilon}{t^{1/2}}\widetilde{\mu}_{\alpha+1}(k,1)\ ,\end{aligned}$$

and furthermore that

$$\begin{aligned}\left|\int_{t-(t-1)^{3/4}}^{t} e^{-|k|(t-s)} q_1(k,s,t)\ ds\right| &\leq \varepsilon\mu_{\alpha+1}(k,t)\int_{t-(t-1)^{3/4}}^{t} e^{-|k|(t-s)}\,|k|\,\frac{t-s}{(\sqrt{s}+\sqrt{t})\sqrt{ts}}ds\\
&\leq \frac{\varepsilon}{t^{3/2}}\mu_{\alpha+1}(k,t)\int_{t-(t-1)^{3/4}}^{t} e^{-|k|(t-s)}\,|k|\,(t-s)ds\\
&\leq \frac{\varepsilon}{t^{3/2}}\mu_{\alpha+1}(k,t)\int_{t-(t-1)^{3/4}}^{t} ds \leq \frac{\varepsilon}{t^{3/4}}\mu_{\alpha+1}(k,t)\ .\end{aligned}$$

This completes the proof of Proposition 23. ■

6.2 Proof of Theorem 18

Let Ω_- and U_- as defined in Proposition 23, and let

$$\begin{aligned}U(k,t) &= -\frac{i}{k}\Lambda_+\Omega_-(k,t) + U_-(k,t)\ ,\\
V(k,t) &= \Omega_-(k,t) + i\sigma(k)U_-(k,t)\ .\end{aligned}$$

From (27), and using that $|\Lambda_-/k| < 1$, we find for $t \geq 1$ that

$$\left|-\frac{i}{k}\Lambda_-\omega_+\right| \leq \frac{\varepsilon}{t^2}\frac{1}{\Delta}\frac{1}{\Lambda_+}\mu_\alpha(k,t) \leq \frac{\varepsilon}{t^{3/2}}\mu_{\alpha+1}(k,t)\ .$$

Using in addition the inequalities (80)-(82) we therefore get for u using (15), that

$$\begin{aligned}|(u-U)(k,t)| &\leq \frac{\varepsilon}{t^{3/2}}\mu_{\alpha+1}(k,t) + \frac{\varepsilon}{t^{1/2}}\mu_{\alpha+1}(k,t)\\
&\quad + \frac{\varepsilon}{t^{3/4}}\mu_{\alpha+1}(k,t) + \frac{\varepsilon}{t^{1/2}}\widetilde{\mu}_{\alpha+1}(k,t)\\
&\leq \frac{\varepsilon}{t^{1/2}}\mu_{\alpha+1}(k,t)\ ,\end{aligned}\tag{86}$$

the worst bound being the contribution from ω_-. Similarly we get for v using (15) and the inequalities (79), (81) and (82), that

$$\begin{aligned}|(v-V)(k,t)| &\leq \frac{\varepsilon}{t^{3/2}}\mu_{\alpha+1}(k,t) + \frac{\varepsilon}{t}\mu_{\alpha+1}(k,t)\\
&\quad + \frac{\varepsilon}{t^{3/4}}\mu_{\alpha+1}(k,t) + \frac{\varepsilon}{t^{1/2}}\widetilde{\mu}_{\alpha+1}(k,t)\\
&\leq \frac{\varepsilon}{t^{3/4}}\mu_{\alpha+1}(k,t) + \frac{\varepsilon}{t^{1/2}}\widetilde{\mu}_{\alpha+1}(k,t)\ ,\end{aligned}\tag{87}$$

the worst bounds being here the contributions from u_+ and u_-. The bounds (86) and (87) imply in direct space for $\alpha > 0$, that except for the terms U and V, respectively, contributions to u and v are bounded by $\mathcal{O}(1/x)$ and $\mathcal{O}(1/x^{5/4})$, uniformly in y. Since, moreover,

$$\lim_{t\to\infty} -\frac{i}{k/t^{1/2}}\Lambda_+(\frac{k}{t^{1/2}})\Omega_-(\frac{k}{t^{1/2}},t) = \left(-i\partial_k\omega_-^*(0) + i\int_1^\infty \partial_k q(0,s)\ ds\right) e^{-k^2} \ , \tag{88}$$

$$\lim_{t\to\infty} t^{1/2}\Omega_-(\frac{k}{t^{1/2}},t) = -\left(-i\partial_k\omega_-^*(0) + i\int_1^\infty \partial_k q(0,s)\ ds\right)(-ik)\, e^{-k^2} \ , \tag{89}$$

$$\lim_{t\to\infty} U_-(\frac{k}{t},t) = \left(u_-^*(0) - \frac{i}{2}\int_1^\infty \partial_k q(0,s)\ ds\right) e^{-|k|} \ , \tag{90}$$

and furthermore

$$\left|-\frac{i}{k/t^{1/2}}\Lambda_+(\frac{k}{t^{1/2}})\Omega_-(\frac{k}{t^{1/2}},t)\right| \leq \varepsilon\mu_{\alpha+1}(k,1) \ ,$$

$$\left|t^{1/2}\Omega_-(\frac{k}{t^{1/2}},t)\right| \leq \varepsilon\mu_{\alpha+1}(k,1) \ ,$$

$$U_-(\frac{k}{t},t) \leq \varepsilon\mu_{\alpha+1}(k,1) \ ,$$

uniformly in $t \geq 1$, it follows by the Lebesgue dominated convergence theorem that the convergence in (88)-(90) is not only pointwise but also in $L^1(\mathbf{R})$. The equalities (69), (70) and (75) now follow by using in addition the identities for $u(0,1)$ and $v(0_+,1)$ established in the proof of Proposition 14. The existence of the integrals in (73), (74) and (75) in the (C,δ)-sense (see Remark 19) follows by using that u and U are by definition continuous functions of $k \in \mathbf{R}$ and that v and V are by definition continuous functions of $k \in \mathbf{R}\setminus\{0\}$. This completes the proof of Theorem 18.

6.3 Appendix

Proposition 24 (Improved version of Proposition 7) *Let $\alpha' \geq \beta' \geq \gamma' \geq 0$. Then, for all $t \geq 1$ and $k \in \mathbf{R}$,*

$$\frac{1}{1+|k|^{\alpha'}} e^{\Lambda_- \frac{t-1}{2}} \left|\Lambda_-\right|^{\beta'} \left(\frac{t-1}{t}\right)^{\gamma'} \leq \text{const.}\frac{1}{t^{\beta'}}\frac{1}{1+\left(|k|\, t^{1/2}\right)^{\alpha'-\beta'+\gamma'}} \ . \tag{91}$$

Proof. For $1 \leq t < 2$ we have that

$$\begin{aligned}
\frac{1}{1+|k|^{\alpha'}} e^{\Lambda_- \frac{t-1}{2}} \left|\Lambda_-\right|^{\beta'} \left(\frac{t-1}{t}\right)^{\gamma'} &\leq \text{const.}\ \frac{1}{1+|k|^{\alpha'}} e^{\Lambda_- \frac{t-1}{2}} \left|\Lambda_-\,(t-1)\right|^{\gamma'}\ \ \left|\Lambda_-\right|^{\beta'-\gamma'} \\
&\leq \text{const.}\ \frac{1}{1+|k|^{\alpha'}}\ \ \left|\Lambda_-\right|^{\beta'-\gamma'} \\
&\leq \text{const.}\ \frac{1}{1+|k|^{\alpha'-\beta'+\gamma'}} \\
&\leq \text{const.}\frac{1}{t^{\beta'}}\frac{1}{1+\left(|k|\, t^{1/2}\right)^{\alpha'-\beta'+\gamma'}} \ ,
\end{aligned}$$

as claimed, and for $t > 2$ we use that

$$
\begin{aligned}
&\left(1+\left(|k|\, t^{1/2}\right)^{\alpha'-\beta'+\gamma'}\right) e^{\Lambda_- \frac{t-1}{2}} \, |\Lambda_- t|^{\beta'} \left(\frac{t-1}{t}\right)^{\gamma'} \\
&\leq \left(1+\left(|k|\, t^{1/2}\right)^{\alpha'}\right) e^{\Lambda_- \frac{t-1}{2}} \, |\Lambda_- t|^{\beta'} \left(\frac{t-1}{t}\right)^{\gamma'} \\
&\leq \text{const.} \left(1+\left(|k|\, t^{1/2}\right)^{\alpha'}\right) e^{\Lambda_- \frac{t}{4}} \, |\Lambda_- t|^{\beta'} \\
&\leq \text{const.} \left(1+\frac{|k|^{\alpha'}}{|\Lambda_-|^{\alpha'/2}} \, |\Lambda_- t|^{\alpha'/2} \, |\Lambda_- t|^{\beta'} \, e^{\Lambda_- \frac{t}{4}}\right) \\
&\leq \text{const.} \left(1+\frac{|k|^{\alpha'}}{|\Lambda_-|^{\alpha'/2}}\right) \\
&\leq \text{const.} \left(1+|k|^{\alpha'/2}\right) \leq \text{const.} \left(1+|k|^{\alpha'}\right) \ ,
\end{aligned}
$$

and (91) follows. ■

References

[1] P. Wittwer: On the structure of stationary solutions of the Navier-Stokes equations. To appear in Comm. Math. Phys. (For a preprint see mp-arc, 01-102).

[2] G. K. Batchelor: An introduction to fluid dynamics. Cambridge University Press, 1967.

[3] E. C. Titchmarsh: Theory of Fourier Integrals. Oxford, at the Clarendon press, 1937.